Koopman-based Data-Driven Control of Nonlinear Systems

Robin Strässer, Julian Berberich, Frank Allgöwer

University of Stuttgart, Institute for Systems Theory and Automatic Control,
Pfaffenwaldring 9, 70569 Stuttgart, Germany
robin.straesser@ist.uni-stuttgart.de

Data-driven analysis and control methods of dynamical systems have gained a lot of interest in recent years. While the class of linear systems is well studied, theoretical results for nonlinear systems are still rare. Our approach relies on the Koopman operator, which is a linear but infinite-dimensional operator lifting the nonlinear system to a higher-dimensional space.

In this poster, we present a data-driven controller design method with stability guarantees for *unknown* discrete-time control-affine nonlinear systems

$$x_{k+1} = f(x_k) + g(x_k)u_k, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $f : \mathbb{R}^n \to \mathbb{R}^n$, and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$. Relying on Koopman operator theory [1], we define a nonlinear lifting $z = \Phi(x)$ with $\Phi : \mathbb{R}^n \to \mathbb{R}^N$, $N \geq n$, to derive a lifted bilinear representation of the underlying system described by

$$z_{k+1} = Az_k + B_0 u_k + \begin{bmatrix} B_1 & \cdots & B_m \end{bmatrix} (u_k \otimes z_k) + \varepsilon_k, \tag{2}$$

where $A \in \mathbb{R}^{N\times N}$, $B_0 \in \mathbb{R}^{N\times m}$, $B_j \in \mathbb{R}^{N\times N}$, $j = 1, ..., m$, and $\otimes$ denotes the Kronecker product. The nominal bilinear representation is generally only an approximation of the true underlying nonlinear system, which is why we include the modeling error ε to obtain an equivalent characterization. Since the underlying system dynamics are unknown and only measured data samples $\{x_k^{\mathrm{d}}\}_{k=0}^{L}$, $\{u_k^{\mathrm{d}}\}_{k=0}^{L-1}$ are available, we estimate the system matrices of the bilinear system from data. To this end, we solve the least-squares problem

$$\begin{aligned} \min_{A,B_0,B_1,\ldots,B_m} \Big\| &\begin{bmatrix} \Phi(x_1^{\mathrm{d}}) & \cdots & \Phi(x_L^{\mathrm{d}}) \end{bmatrix} \\ &- A \begin{bmatrix} \Phi(x_0^{\mathrm{d}}) & \cdots & \Phi(x_{L-1}^{\mathrm{d}}) \end{bmatrix} - B_0 \begin{bmatrix} u_0^{\mathrm{d}} & \cdots & u_{L-1}^{\mathrm{d}} \end{bmatrix} \\ &- \begin{bmatrix} B_1 & \cdots & B_m \end{bmatrix} \begin{bmatrix} (u_0^{\mathrm{d}} \otimes \Phi(x_0^{\mathrm{d}})) & \cdots & (u_{L-1}^{\mathrm{d}} \otimes \Phi(x_{L-1}^{\mathrm{d}})) \end{bmatrix} \Big\| \end{aligned} \tag{3}$$

to obtain the ℓ_2-optimal estimate based on the given data. Additionally, we account for the modeling error ε using a Lipschitz-continuity argument. Then, we obtain a linear matrix inequality based controller design procedure guaranteeing robust local stability for the resulting bilinear system for all errors consistent with the Lipschitz bound [2]. In particular, we build our approach on the linear fractional representation

$$\begin{bmatrix} z_+ \\ u \end{bmatrix} = \begin{bmatrix} A & B_0 & \begin{bmatrix} B_1 & \cdots & B_m \end{bmatrix} & I \\ 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ u \\ w \\ \varepsilon \end{bmatrix}, \tag{4a}$$

$$w = (I_m \otimes z)u \tag{4b}$$

for all z satisfying

$$\begin{bmatrix} z \\ 1 \end{bmatrix}^\top \begin{bmatrix} Q_z & S_z \\ S_z^\top & R_z \end{bmatrix} \begin{bmatrix} z \\ 1 \end{bmatrix} \geq 0, \qquad Q_z \prec 0, R_z \succ 0, \tag{5}$$

and the control law $u = Kz + K_w w$. More precisely, we interpret the bilinearity as a measurable uncertainty which is known to be bounded and use ideas from gain-scheduling [3] for a robustly stabilizing controller design. Thus, the proposed controller also stabilizes the underlying nonlinear system.

Finally, we apply the developed design method to the nonlinear Van der Pol oscillator to illustrate our theoretical findings. Compared to a Koopman-based controller design proposed in [4], our result clearly outperforms their closed-loop behavior and successfully stabilizes the nonlinear system (see Figure 1).

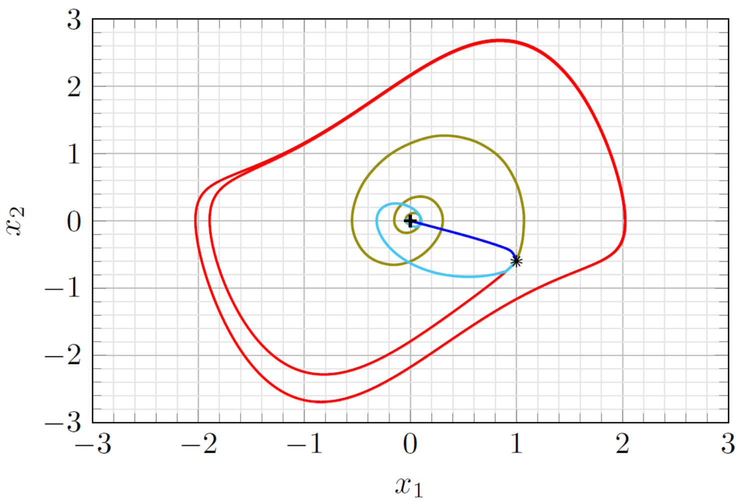

Figure 1: Open-loop (—) and closed-loop trajectories of the Van der Pol oscillator using our proposed approach with initial condition ($*$) and origin ($+$). The shown closed-loop trajectories are obtained for two different controllers u_1 (—), u_2 (—), and, as comparison, the controller designed in [4] (—).

References

[1] B. O. Koopman, "Hamiltonian systems and transformation in Hilbert space," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 17, no. 5, p. 315, 1931.

[2] R. Strässer, J. Berberich, and F. Allgöwer, "Robust data-driven control for nonlinear systems using the koopman operator," *submitted to IFAC World Congress*, 2023.

[3] C. W. Scherer, "LPV control and full block multipliers," *Automatica*, vol. 37, no. 3, pp. 361–375, 2001.

[4] S. Sinha, S. P. Nandanoori, J. Drgona, and D. Vrabie, "Data-driven stabilization of discrete-time control-affine nonlinear systems: A Koopman operator approach," in *Proc. European Control Conference (ECC)*, 2022.