Solvers for Hybrid Modelling in Porous Media – Design, Implementation, Analysis

Cedric Riethmüller[1], Dominik Göddeke[1,2]

[1]) University of Stuttgart, Institute of Applied Analysis and Numerical Simulation, Allmandring 5b, 70569 Stuttgart, Germany
cedric.riethmueller@ians.uni-stuttgart.de
[2]) University of Stuttgart, Stuttgart Center for Simulation Science (SC SimTech)

We explore solver aspects for PDE- and data-based numerical schemes that provide discretisations for coupled multi-phase flow problems. This includes free-flow/porous-media-flow coupling and coupled porous media domains with two-phase flow hierarchies. The mathematical models are fundamental for large-scale simulations in the context of geothermal energy storage and production as well as other green-energy settings. Many current research efforts are directed towards the modelling of such phenomena and their numerical treatment, and substantial progress has been made in the past decade. However, less focus has been on efficient numerical techniques to simulate them. This leads to a bottleneck when real-world scenarios are considered. We develop solvers that combine physics- and data-based preconditioning techniques and pursue both concepts based on the monolithic and the partitioned/splitting paradigms. Preconditioners are tailored to the differential operators in the underlying equations in a block-like fashion. We strive for data-driven techniques to act as surrogate models or determine numerical parameters optimising the performance of the discretisation on the fly.

Here, we focus on two different multi-phase flow problems. Firstly, we discuss free-flow/porous-media-flow coupling as depicted in Fig. 1. The underlying model comprises the Stokes equations and Darcy's law in the respective domains along with boundary and coupling conditions [4].

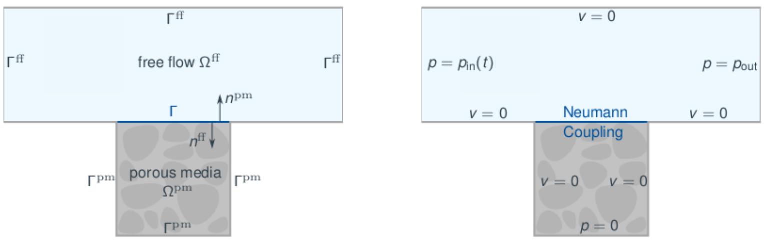

Figure 1: The illustration depicts a coupled two-domain problem where a free-flow over a porous medium is investigated.

Secondly, we consider precipitation and dissolution effects [3]. We investigate a two-phase flow problem, comprising the Navier-Stokes equations for evaluating the flow field and the Cahn-Hilliard equation for calculating the evolving diffuse interface between the fluid and solid phases as illustrated in Fig. 2.

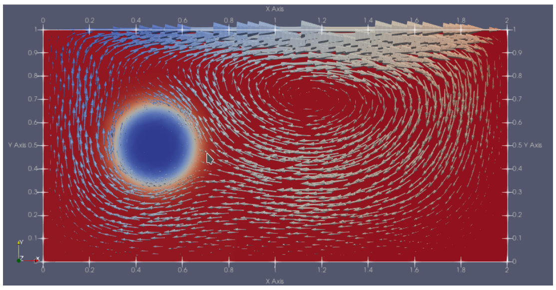

Figure 2: The illustration depicts the simulation result of the two-phase flow model problem where the red part represents the fluid and the blue one the solid phase. The color transition from red to blue indicates the diffuse interface. The arrows illustrate the velocity field.

To solve the discrete, non-linear problems arising from a spatio-temporal discretisation, Newton's method is applied. A key focus is on the solution of the arising large, sparse and ill-conditioned linear systems. Using problem-adapted [1] and physics-inspired (non-)linear preconditioning [2], these systems can be solved much more efficiently than by using stock techniques. We discuss both monolithic block-preconditioned approaches as well as partitioned coupling approaches. The poster will give an overview of the described topics.

References

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