HIGHER-ORDER SEMI-EXPLICIT TIME INTEGRATION METHODS FOR POROELASTICITY PROBLEMS

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The linear Biot poroelasticity model with and without the quasi-static assumption give rise to coupled elliptic–parabolic and coupled hyperbolic–parabolic partial differential equations (PDEs), respectively. Semi-explicit time integration methods for these coupled PDEs enable decoupled solves with efficient solvers and black-box preconditioners for the subsystems. Hence, this reduces the computational burden that is associated with solving the coupled PDEs monolithically. The design of these specialized semi-explicit time integration methods is based on constructing well-posed PDEs containing time-delays, that naturally decouple the original PDE and approximate its solution up to a given order. An appropriate order implicit Runge–Kutta time discretization then yields the semi-explicit methods which are multi-step by construction.

We discuss the key differences between well-posed delay systems that approximate the elliptic–parabolic PDEs and the hyperbolic–parabolic PDEs, respectively. This include the conditional stability of the delay systems or the restriction on the time-delay with respect to the parameters of the PDEs. Moreover, we present the ideas of the stability and the convergence analysis for higher-order semi-explicit methods and demonstrate the same with numerical examples.

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