Construction of composite difference schemes with a given order of accuracy

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The process of constructing new numerical algorithms is usually based on the implementation of the two most common sequential stages [1], [2], consisting in the compilation of discrete (difference) approximations of the equations and checking the a priori qualitative characteristics of these approximations, mainly such as stability, convergence and order of accuracy of the obtained difference schemes. The problems related to the implementation of the first stage are considered and substantiated in detail in [4], [3], where the difference schemes of one-step and multistep block methods oriented to the solution of the Cauchy problem for ordinary differential equations (ODE) are given. In [5] differential schemes of Bickart-type methods are proposed, which, in addition, provide an opportunity to control the integration step. Also, in [4], [6] the use of collocation difference schemes in the numerical solution of evolutionary partial differential equations was considered.

In this work we investigated methods of compositing difference approximations to form single-step and multi-step parallel difference schemes of a given order, focused on the numerical solution of the Cauchy problem both for ordinary differential equations and for evolutionary differential equations, which can be reduced to the solution of ODE, for example, by the method of line [7].

In order to analyze the stability, we investigated the a priori estimates of the solution of the difference problem in the light of the available initial data, that is, the stability analysis on the initial data and on the right-hand side [1], [4], was carried out. The main result of the research carried out in this direction was the determination of a subset of stable (absolutely orA-α) difference schemes of the primary family.

Then, among the found subset of stable schemes, schemes with a given accuracy, computation volume, and other desired properties and parameters were determined [4], [3]. Among the main problems arising at this stage, it is necessary to identify the achievable order of accuracy of a difference scheme for different classes of problems, to develop schemes for solving a wide class of problems with a certain guaranteed accuracy, to build schemes with improved accuracy in narrower classes of problems, and to formulate general principles for building stable difference schemes with minimal indices of implementation labor intensity.

The construction of discrete approximations is carried out by limiting the estimate on the approximation error of the difference method taking into account the dimensions of the computational and/or reference blocks. Approaches based on the use of an integro-interpolation method or on the ratio of Taylor series expansions of the exact and approximate solutions were used. This allowed varying the order of error, departing from the maximum possible one on a fixed set of knots, but ensuring absolute or A-α stability of numerical solutions. In the paper it is proved that one-step schemes of the maximum order of approximation are absolutely stable. Absolute stability of single-step schemes is retained even when the dimensionality of computational blocks is increased. At the same time, multistep methods with maximal approximation orders are unstable. For such cases, ensuring the condition of absolute or A-α stability of the methods was carried out by decreasing the maximum possible order of approximation.

We also investigated difference compositions with high order derivatives based on Hermite polynomials. The introduction of additional derivatives in the difference schemes contributed to a significant increase in the order of approximation, without leading to an increase in the dimensionality of the system of equations, which is formed for each new computational block. The difference compositions with high order derivatives also allowed to equalize the order of approximation for all computational points in the block.

The proposed approaches to forming compositions are universal for obtaining different types of difference equations. Computational formulas for multi-point difference equations obtained on the basis of these approaches have less computational complexity and are oriented toward implementation in computing systems with parallel architecture. The proposed difference schemes allow increasing the efficiency of parallel simulation and optimization of dynamic processes described by evolution equations with concentrated and distributed parameters.

To form difference relations with a given dimensionality of computational and reference blocks a program complex was created that allows to find coefficients of transition matrices, to determine approximation orders in computational points and to estimate stability of obtained schemes.

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