A Generalized Framework of Neural Networks for Hamiltonian Systems

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When solving Hamiltonian systems using numerical integrators, preserving the symplectic structure is crucial [4]. At the same time, solving chaotic problems requires integrators to approximate the trajectories with extreme precision. This can be very computationally expensive. However, for example in [1] it was shown that a neural network can be a viable alternative to numerical integrators. Offering high accuracy solutions for the chaotic N-body problem many orders of magnitudes faster.

To understand when it is useful to add physics constraints into neural networks, we analyze three well-known neural network topologies that include a symplectic structure inside the NN architecture [3, 6, 2]. Between these neural network topologies many similarities can be found [5]. This allows us to formulate a generalized framework for these topologies. With the new framework, we can find novel topologies by transitioning between the established ones.

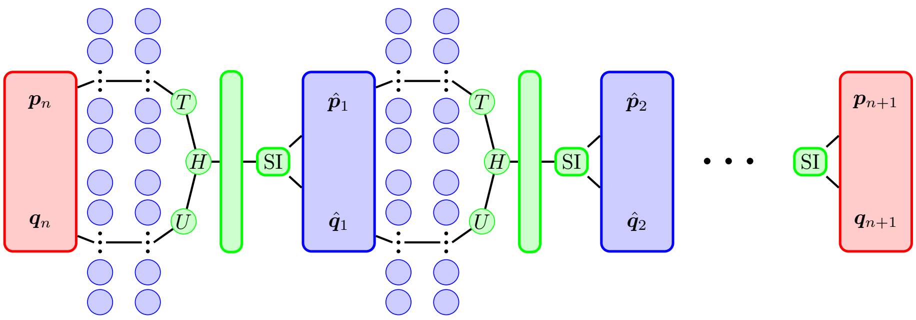

Figure 1: A visualization of the generalized framework of neural networks for Hamiltonian systems. (GHNN)

We compare these new Generalized Hamiltonian Neural Networks (GHNN) against the already established SympNets and HénonNets and physics-unaware multilayer perceptrons. This comparison is performed with data from a pendulum, a double pendulum and a gravitational three-body problem. A special focus lies on the generalization capabilities outside the training data. We found that the GHNN outperforms all other neural network architectures.

References

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