Dictionary-based Online-adaptive Structure-preserving Model Order Reduction for Parametric Hamiltonian Systems

Robin Herkert[1], Patrick Buchfink[1], Bernard Haasdonk[1]

[1]) Institute of Applied Analysis and Numerical Simulation, University of Stuttgart, Germany
{robin.herkert, patrick.buchfink, bernard.haasdonk}@ians.uni-stuttgart.de

The success of classical Model Order Reduction (MOR) is based on the assumption that the solution manifold of dynamical systems can be approximated well in a low-dimensional subspace. However, some problems, especially problems with slowly decaying Kolmogorov-n-widths (e.g. transport problems) do not allow such a low-dimensional approximation. This may lead to reduced order bases that are too large and result in an insufficient speed up compared to the high-order simulation. In general, the development of online-efficient MOR for these problems is still strongly investigated.

Additionally, models may posses the form of a parametric Hamiltonian systems.

$$\dot{\mathbf{x}}(t) = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{pmatrix} \nabla_{\mathbf{x}} \mathbf{H}(\mathbf{x}(t), \boldsymbol{\mu}) = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{pmatrix} \mathbf{L}(\boldsymbol{\mu})\mathbf{x}(t) + \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{pmatrix} \mathbf{f}_{\mathrm{nl}}(\mathbf{x}(t), \boldsymbol{\mu}). \tag{1}$$

The mathematical structure of a Hamiltonian system ensures conservation of energy and, under mild assumptions, stability properties and should therefore be preserved during MOR. Classical MOR fails to preserve this Hamiltonian structure. Structure-preserving, symplectic MOR for Hamiltonian systems [3] is based on the generation of a symplectic basis and a projection with the so-called symplectic inverse of the basis.

In our work, we merge ideas from dictionary-based MOR with structure-preserving MOR for Hamiltonian systems. The idea of a dictionary-based approach is that the basis is computed in the online-phase during the time stepping of the reduced simulation and is thus parameter- and time-dependent. Therefore, it can be expected that the online-adaptive basis may be much smaller than a classical reduced order basis. This could lead to a higher speed-up compared to a non-adaptive approach, depending on how efficient the basis changes and basis computations can be performed.

For the basis computation, a dictionary of state snapshots

$$\mathbf{D}_{\mathrm{X}} := \{x_1, ..., x_{N_{\mathrm{X}}}\} \subset \mathbb{R}^{2N}$$

is constructed in the offline-phase. In order to efficiently treat non-linearities in the right-hand side of the ODE system in combination with the DEIM-algorithm, dictionaries of non-linearity snapshots $\mathbf{D}_{\mathrm{F}}$ and DEIM-indices $\mathbf{D}_{\mathrm{P}}$ are computed:

$$\mathbf{D}_{\mathrm{F}} := \{\mathbf{f}_{\mathrm{nl}}(x_1), ..., \mathbf{f}_{\mathrm{nl}}(x_{N_{\mathrm{X}}})\} \subset \mathbb{R}^{2N} \quad \text{and} \quad \mathbf{D}_{\mathrm{P}} := \{i_1, ..., i_{N_{\mathrm{P}}}\} \subset \{1, ..., 2N\}.$$

In Fig. 1, we present a worflow sketch of the online-phase of our algorithm. For a given parameter $\boldsymbol{\mu}$ and start time t_0, we start with the selection of snap-

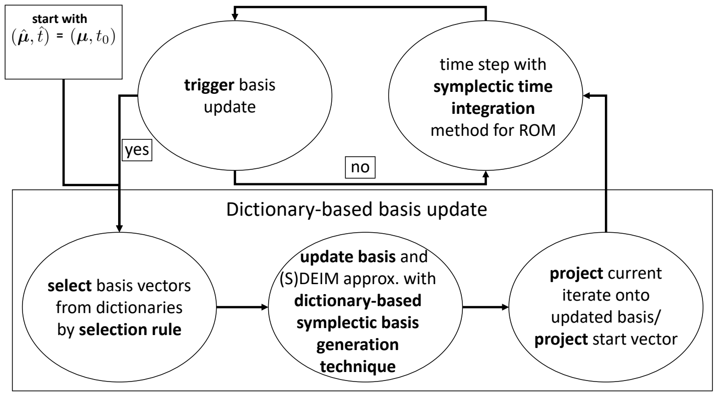

Figure 1: Workflow sketch

shots from the dictionary. The selection rule chooses snapshots according to distances in the parameter-time-space. For that purpose we extend ideas from [4] to the instationary case. Then, the basis and SDEIM approximation are computed using the newly developed dictionary-based methods and the start vector is projected. The projections with the basis matrices are performed in an implicit, online-efficient manner, i.e. independently from the state dimension. After that, the time stepping of the reduced system is executed until a basis update is triggered. After a basis update has been performed, the current state is projected from the old to the new reduced space and the time-stepping is continued. These steps are repeated until the final simulation time t_{End} is reached.

Numerical experiments are performed on a linear and a non-linear wave-equation model. The influence of the basis changes on the conservation of the Hamiltonian is analyzed.

References

[1] Buchfink, P. and Bhatt, A. and Haasdonk, B. 2019. Symplectic Model Order Reduction with Non-Orthonormal Bases. *Mathematical and Computational Applications, 24(2):* p.43

[2] Herkert, R. 2022. Dictionary-based Online-adaptive Structure-preserving Model Order Reduction for Parametric Hamiltonian Systems. Master Thesis

[3] Peng, L., and Mohseni, K. 2016. Symplectic Model Reduction of Hamiltonian Systems. *SIAM Journal on Scientific Computing, 38(1):* pp. A1–A27

[4] Maday, Y. and Stamm, B. 2013. Locally adaptive greedy approximations for anisotropic parameter reduced basis spaces. *SIAM Journal on Scientific Computing, 35(6):* pp. A2417–A2441.