A Sampling-free Statistical Finite Element Method in Computational Mechanics

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We explore integrating sensor data and computational predictions in mechanics using the statistical finite element method (statFEM). StatFEM has recently been introduced as a physics-constrained data regression technique to infer the true behavior of a physical system from indirect measurements [1]. We adopt a mechanics perspective, where the material coefficients exhibit uncertainties, and the model predicts the displacement field. Following the statFEM paradigm, we first conduct a probabilistic finite element analysis with a stochastic material parameter and the Polynomial Chaos method [2]. Then, we employ a Gaussian process model to explicitly consider model-reality error and update the hyperparameters alongside the displacement field in a Bayesian framework.

In contrast to Bayesian parameter updating methods [3], statFEM does not update the stochastic material coefficients. Instead, we assume that a stochastic calibration step has already been performed, and our focus is on online prediction based on sensor and simulation data. Specifically, the stochastic material coefficient is employed in a mechanical model to compute a physically motivated prior for updating and predicting the displacements [4].

In this study, we formulate the three components of the statistical generating model, namely the displacement, the model-reality mismatch, and the noise, as three independent random fields in their suitable Polynomial Chaos bases. A crucial challenge is to combine these Chaos bases into a unified basis to form an extended Polynomial Chaos-based statistical generating model. We then utilized the Kalman filter to update the PC coefficients of the stochastic displacement field, drawing on ideas from [3]. With these coefficients, we can efficiently compute the mean value and covariance of the posterior displacement without sampling. It has to be noted that the hyperparameters are not the scaling and lengthscale parameters, as in [1], but the PC coefficients of the model-reality mismatch, described as a non-Gaussian random field. As a numerical example, we selected a two-dimensional Timoshenko-Goodier cantilever beam with an uncertain Young's modulus described by the Karhunen-Loève expansion as a lognormal random field. The Polynomial Chaos-based statFEM approach offers several benefits, including the ability to perform online data updates, making it a promising method for digital twinning with uncertainty quantification. In Fig. 1, we can see how statFEM can develop a so-called "statistical digital twin" for a wind turbine in light of observation data.

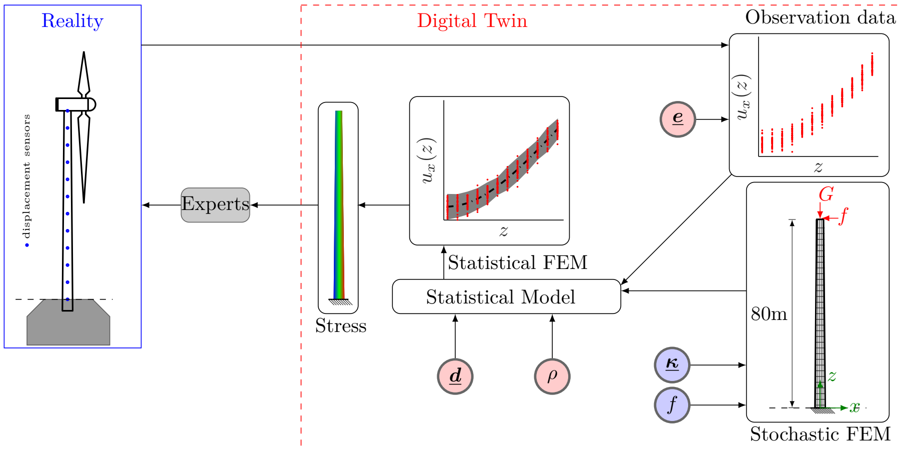

Figure 1: Statistical Digital Twin.

References

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