Gaussian Process Techniques for Differential Equations

Aleksandar Arsenijevic

University of Stuttgart, Institute for Stochastics and Applications (ISA),
Pfaffenwaldring 57, 70569 Stuttgart, Germany
aleksandar.arsenijevic@mathematik.uni-stuttgart.de

Gaussian Processes for Machine Learning (GP4ML) are an established kernel-based Bayesian machine learning method that has also been considered in the context of differential equations. The central point of GP4ML is the update step. Here, information in form of equations or observations are used to transform, through conditioning, the prior Gaussian process into an a-posteriori Gaussian process. From a mathematical point of view, however, the application of GP4ML for differential equations is not sufficiently justified, in particular if it comes to infinite rank conditioning. The latter notion of conditioning, however, is somewhat natural either as an idealized limit of existing methods or as a methodologically independent approach of its own. Our goal is to address these shortcomings by establishing a rigorous and general conditioning theory for GP4MLs and to apply this theory in the context of differential equations. One particular focus lies on approximations with finite rank conditionings by employing for example martingale techniques.

References

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